Generalized Series Imaging with Multiple References

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Abstract

Many imaging applications require the acquisition of a time series of images. In conventional Fourier transform-based imaging methods, each of these images is acquired independently. As a result, one often has to sacrifice spatial resolution for temporal resolution. To address this problem, this paper extends a generalized series imaging method so that multiple references can be used to achieve high spatiotemporal resolution. Application results are also presented to illustrate its effectiveness for high-resolution dynamic imaging.

1. Introduction

Magnetic resonance imaging (MRI) is usually implemented as a Fourier transform-based technique. As such, the imaging equation is in the form of a Fourier integral. For notational convenience, we define here the dynamic imaging problem as the acquisition of a sequence of Q images, denoted as $I_1(x), I_2(x), \cdots, I_Q(x)$, each of which is a snapshot of a time-varying image function $I(x,t)$. Conventionally, Q data sets in k-space, $d_1(k), d_2(k), \cdots, d_Q(k)$, are acquired independently. Assume that N sample data points (or encodings) are collected for each of these data sets at $k = n\Delta k$, $-N/2 \leq n < N/2$. The spatial resolution of $I_q(x)$ is limited to $1/(N\Delta k)$, whereas the temporal resolution is NT_R with T_R being the repetition time. Therefore, high spatial resolution requires a large N, which means poor temporal resolution.

To overcome this problem, several data-sharing methods have been proposed for efficient dynamic imaging [1–6], two examples of which are the Keyhole and RIGR (Reduced-encoding Imaging by Generalized-series Reconstruction) techniques [1–3]. This paper presents an extension on the RIGR technique so that multiple references can be used to achieve high spatiotemporal resolution.

2. Generalized Series Imaging

2.1 The generalized series model

The generalized series (GS) model is a general mathematical model developed for constrained image reconstruction. In this model, an image function is represented as

$$I_{\text{gs}}(x) = \sum_n c_n \varphi_n(x), \tag{1}$$

where $\varphi_n(x)$ are basis functions adaptively chosen according to the conditions of a particular application problem. Of special interest to dynamic imaging is the class of basis functions given by

$$\varphi_n(x) = \mathcal{C}(x) e^{i2\pi n\Delta kx}, \tag{2}$$

where the constraint function $\mathcal{C}(x)$ is normally chosen to absorb available *a priori* information. Many desirable properties of these basis functions have been discussed in the context of constrained spectroscopic imaging [7] or image reconstruction in general [8]. The optimality of these basis functions can also be justified from the principle of minimum cross-entropy [9]. After the basis functions $\varphi_n(x)$ are chosen, the series coefficients can be determined by solving a set of linear equations. More specifically, forcing $I_{\text{gs}}(x)$ to be data-consistent yeilds

$$d(m\Delta k) = \sum_{n=-N/2}^{N/2-1} c_n d_0[(m-n)\Delta k] \tag{3}$$

for $-N/2 \leq m \leq N/2-1$, where $d_0(k)$ is the Fourier transform of $\mathcal{C}(x)$. Equation (3) can be solved efficiently using, for example, the Levinson algorithm [10].

3. The RIGR Technique

The basic generalized series dynamic imaging scheme (or RIGR) is characterized by the acquisition of one high-resolution reference image and a series of low-resolution

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dynamic data sets. The number of phase encodings for the reference image is chosen to satisfy the spatial resolution requirement, whereas the number of phase encodings for each dynamic data set is chosen to give the desired temporal resolution.

Reconstruction of the dynamic images is accomplished by directly using the generalized series model. Specifically, let $I_{\text{ref}}(x)$ be the high-resolution reference image. The basis functions are constructed as

$$\varphi_n(x) = |I_{\text{ref}}(x)|e^{i2\pi n\Delta kx}. \tag{4}$$

The series coefficients c_n are then determined as described in Section 2.

A notable limitation with the basic RIGR imaging method is due to the fact that the GS model has as many terms as the number of dynamic encodings and, as a result, dynamic information captured by the GS model often does not have the same spatial resolution as the reference image. We demonstrate in this paper that this problem can be effectively overcome if two or multiple references are available.

4. RIGR Imaging with Multiple References

In some dynamic imaging applications, it is possible to acquire two or more high-resolution images at different times of the dynamic imaging process. For example, in contrast-enhanced dynamic imaging of breast cancer, one can acquire a high-resolution pre-contrast image as the baseline reference and another high-resolution reference image when the contrast agent is strongly visible in the image. To reconstruct the dynamic images between the two references, the TRIGR (Two-reference RIGR) method [11] uses the difference image between the two references to shape the basis functions of the GS model. More specifically, let

$$\mathcal{C}(x) = |I_{2,\text{ref}}(x) - I_{1,\text{ref}}(x)| \tag{5}$$

and

$$\Delta d_q(k) = d_q(k) - d_{1,\text{ref}}(k) \tag{6}$$

Dynamic signal changes between the qth data set and the first reference image are expressed in terms of the GS model as

$$\Delta I_q(x) = \mathcal{C}(x) \sum_{n=-M/2}^{M/2-1} c_n e^{i2\pi n\Delta kx} \tag{7}$$

where M is the number of dynamic encodings for each dynamic data set. The c_n can then be determined by fitting the model to the difference data $\Delta d_q(k)$ given in Eq. (6) using the algorithm discussed in Section 2. After $\Delta I_q(x)$ is found, $I_q(x)$ is generated by simply adding $\Delta I_q(x)$ to the reference image. That is,

$$I_q(x) = I_{1,\text{ref}}(x) + \Delta I_q(x). \tag{8}$$

This processing scheme has been demonstrated to be capable of reconstructing dynamic signal variations at high resolution when these variations are of an additive type [11], such as the case with dynamic imaging of injected contrast agents. For multiplicative types of dynamic signal changes, as is the case with diffusion-tensor imaging, this scheme is not effective. To overcome this problem, we have proposed an improved TRIGR reconstruction method, called TRIGR+. This method creates two sets of basis functions based on $I_{1,\text{ref}}(x)$ and $I_{2,\text{ref}}(x)$, respectively. With these basis functions, two reconstructions of $I_q(x)$ are obtained as

$$I_{1,q}(x) = |I_{1,\text{ref}}(x)| \sum_{n=-M/2}^{M/2-1} c_{1,n} e^{i2\pi n\Delta kx}, \tag{9}$$

and

$$I_{2,q}(x) = |I_{2,\text{ref}}(x)| \sum_{n=-M/2}^{M/2-1} c_{2,n} e^{i2\pi n\Delta kx}. \tag{10}$$

The series coefficients $c_{1,n}$ and $c_{2,n}$ are determined by fitting Eqs. (9) and (10) to $\hat{d}_q(m\Delta k) = d_q(m\Delta k)w_m$, respectively, where w_m is chosen according to $w_m = 0.54 + 0.46\cos(2\pi m/M)$, for $-M/2 \le m < M/2$. This filter function serves two purposes: (a) to reduce ringing artifacts due to model truncation, and (b) to create a residual signal $\Delta d_q(m\Delta k) = d_q(m\Delta k)(1 - w_m)$, which can be used to pick up some additional dynamic signal changes missed by both $I_{1,q}(x)$ and $I_{2,q}(x)$.

Note that for a large M, both $I_{1,q}(x)$ and $I_{2,q}(x)$ will reconstruct $I_q(x)$ accurately. This is not the case when M is small. In fact, in this case, $I_{1,q}(x)$ will be different from $I_{2,q}(x)$ although they are both data-consistent. One can argue that $I_{2,q}(x) - I_{1,q}(x)$ will have large values in areas where dynamic signal changes are not well reproduced by $I_{1,q}(x)$ and/or $I_{2,q}(x)$. Therefore, we can use $|I_{2,q}(x) - I_{1,q}(x)|$ to construct another set of basis functions to capture these dynamic signal changes. Specifically, let

$$\Delta I_q(x) = |I_{2,q}(x) - I_{1,q}(x)| \sum_{n=-M/2}^{M/2-1} c_n e^{i2\pi n\Delta kx}. \tag{11}$$

Fitting this model to the residual signal $\Delta d_q(m\Delta k)$ obtained above will not only pick up additional dynamic signal changes but also assure the final reconstruction to be consistent with $d_q(m\Delta k)$ for $-M/2 \le m < M/2$.

Specifically, combining $I_{1,q}(x)$, $I_{2,q}(x)$ and $\Delta I_q(x)$ as follows will yield a data-consistent reconstruction for $I_q(x)$:

$$I_q(x) = \alpha_q I_{1,q}(x) + (1 - \alpha_q)I_{2,q}(x) + \Delta I_q(x). \tag{12}$$

The weighting coefficient α_q can be determined in a variety of ways. For example, assume that $d_{1,\text{ref}}(x)$ and $d_{2,\text{ref}}(x)$

are acquired at $t = t_{1,\mathrm{r}}$ and $t_{2,\mathrm{r}}$, respectively, and that $d_q(k)$ is collected at $t = t_q$ such that $t_{1,\mathrm{r}} \leq t_q \leq t_{2,\mathrm{r}}$. We may set $\alpha_q = (t_{2,\mathrm{r}} - t_q)/(t_{2,\mathrm{r}} - t_{1,\mathrm{r}})$. This scheme is particularly suitable for the case in which dynamic signal changes are roughly a linear function of time. For other models of dynamic signal changes, the α_q can be chosen in a more optimal fashion. Nevertheless, we have found that the simple scheme works reasonably well for a number of problems, including diffusion-tensor imaging and cardiac imaging.

5. Result

Figure 1 illustrates the performance of the proposed method. In this experiment, dynamic signal changes were introduced with variable diffusion weightings using a diffusion-weighted imaging sequence. The images in the top row were reconstructed from 128 phase encodings (each with 128 points) using the conventional Fourier method. The diffusion coefficient map was obtained by fitting a single exponential curve through the image sequence pixel by pixel. When the number of phase encodings is reduced to 8, the Fourier images in the second row show significant blurring as expected. The RIGR method using the same data in the second row plus a high-resolution reference at the beginning of the sequence produced the images in third row. As can be seen, although the diffusion-weighted images appear to be high resolution, the corresponding diffusion map is a low-resolution one, which means that the dynamic signal changes were not produced in high resolution in the diffusion-weighted images. Using the proposed method and the same data in the second row plus two high-resolution references, one at the beginning and the other at the end of the sequence, we obtained the images in the last row. These images demonstrate that the method is able to capture dynamic signal changes at high resolution with only a small number of encodings.

6. Conclusion

An improved method for generalized series imaging has been discussed in this paper. This method enables high-resolution dynamic images to be reconstructed from a reduced number of phase encodings plus two or more references. We have applied the technique to several applications which demonstrate that a 4 to 8 fold improvement in imaging efficiency over the traditional Fourier imaging method can be obtained without a significant loss of image quality. This technique should prove useful for improving temporal resolution for various dynamic imaging experiments, such as dynamic study of injected contrast agents, diffusion-tensor imaging, and cardiac imaging.

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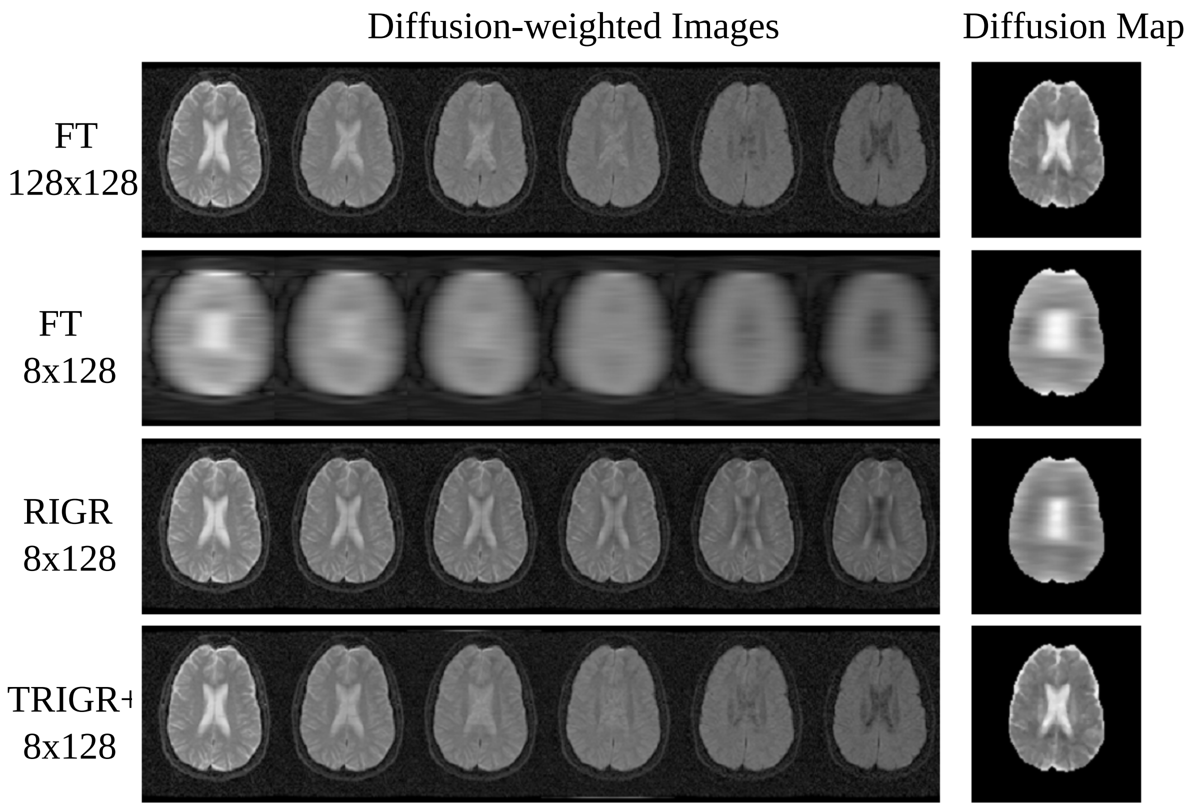

Figure 1. Diffusion-weighted images reconstructed using: (a) the Fourier method with 128×128 encodings; (b) the Fourier method with 8×128 encodings; (c) the RIGR method using the same data in (b) plus a high-resolution reference at the beginning of the sequence; and (d) the TRIGR+ method using the same data in (b) plus two high-resolution references, one at the beginning and the other at the end of the sequence. The diffusion maps were obtained by fitting the diffusion curve pixel-by-pixel through the corresponding image sequences in (a)-(d), respectively.